

For the system shown in Fig. 3.49, x and y denote, respectively, the absolute displacements of the mass m and the end Q of the dashpot c_1 . (a) Derive the equation of motion of the mass m , (b) find the steady-state displacement of the mass m , and (c) find the force transmitted to the support at P , when the end Q is subjected to the harmonic motion $y(t) = Y \cos \omega t$.

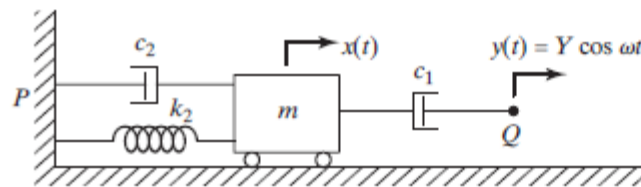
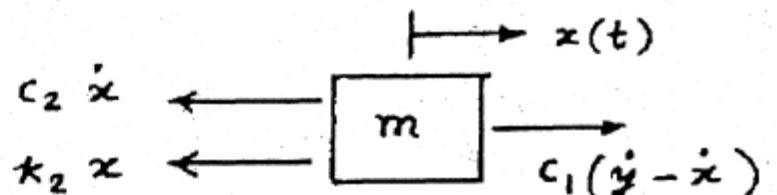


FIGURE 3.49

Solution for the problem of Figure (3.49)

(a) Equation of motion of mass:



$$m\ddot{x} = c_1(\dot{y} - \dot{x}) - c_2\dot{x} - k_2x$$

i.e.,

$$m\ddot{x} + (c_1 + c_2)\dot{x} + k_2x = c_1\dot{y} = -c_1\omega Y \sin \omega t$$

(b)

$$x_p(t) = \frac{-(c_1\omega Y/k_2)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \sin(\omega t - \phi)$$

where $r = \omega/\omega_n$, $\zeta = (c_1 + c_2)\omega/(2rk)$ and $\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$.

(c) steady-state force transmitted to point P:

$$= k_2 x_p + c_2 \dot{x}_p$$

$$= \frac{-(c_1\omega Y)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \left\{ \sin(\omega t - \phi) + \frac{c_2\omega}{k_2} \cos(\omega t - \phi) \right\}$$

Find the steady state response amplitude and the phase angle of the harmonically excited system shown in the figure.

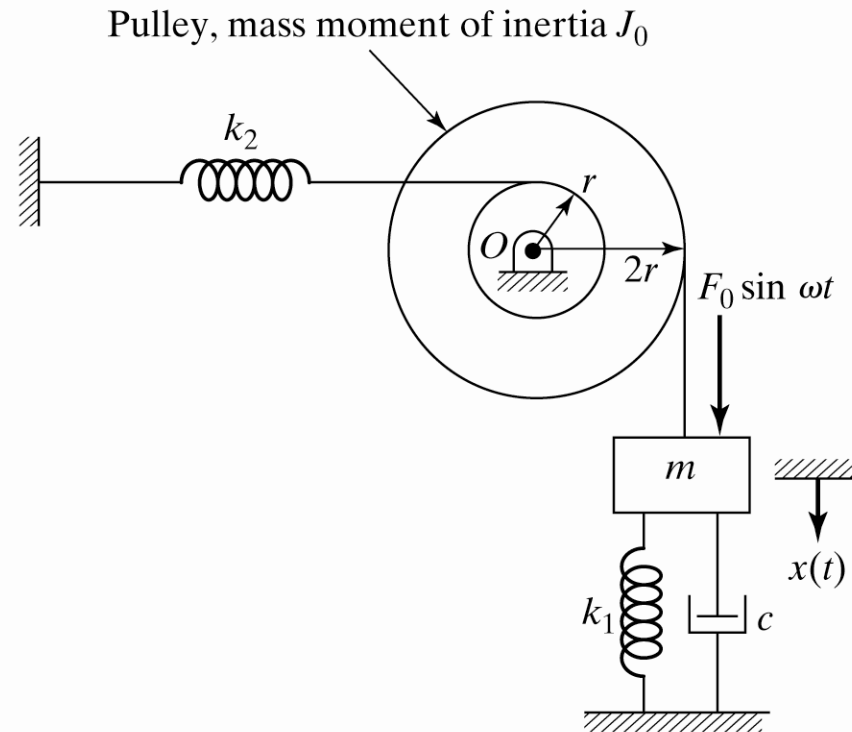


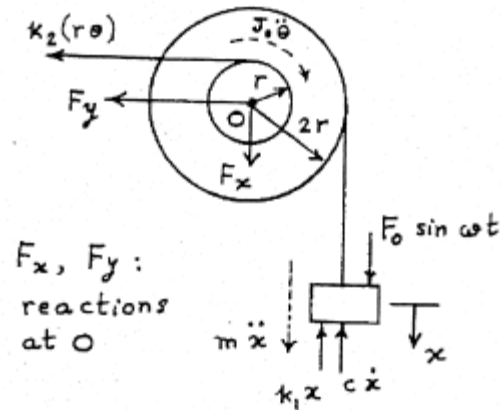
Figure 3.48

$$m\ddot{x} = -k_1 x - c\dot{x} + f_0 \sin(\omega t) - T$$

$$T = -m\ddot{x} - k_1 x - c\dot{x} + f_0 \sin(\omega t)$$

$$-k_2 r^2 \theta + T * 2 * r = J_0 \ddot{\theta}$$

$$-k_2 r^2 \theta + (-m\ddot{x} - k_1 x - c\dot{x} + f_0 \sin(\omega t)) * 2 * r = J_0 \ddot{\theta}$$



Equation of motion for rotation of pulley about O:

$$-k_2 (\theta r) r - J_0 \ddot{\theta} - k_1 x (2r) - c \dot{x} (2r) + F_0 \sin \omega t (2r) - m \ddot{x} (2r) = 0 \quad (1)$$

where $\theta = x/(2r)$. Equation (1) can be rearranged as:

$$\left(\frac{J_0}{2r} + 2m \right) \ddot{x} + 2c r \dot{x} + \left(2k_1 r + \frac{1}{2} k_2 r \right) x = 2r F_0 \sin \omega t \quad (2)$$

For given data, Eq. (2) becomes

$$11 \ddot{x} + 50 \dot{x} + 112.5 x = 5 \sin 20 t \quad (3)$$

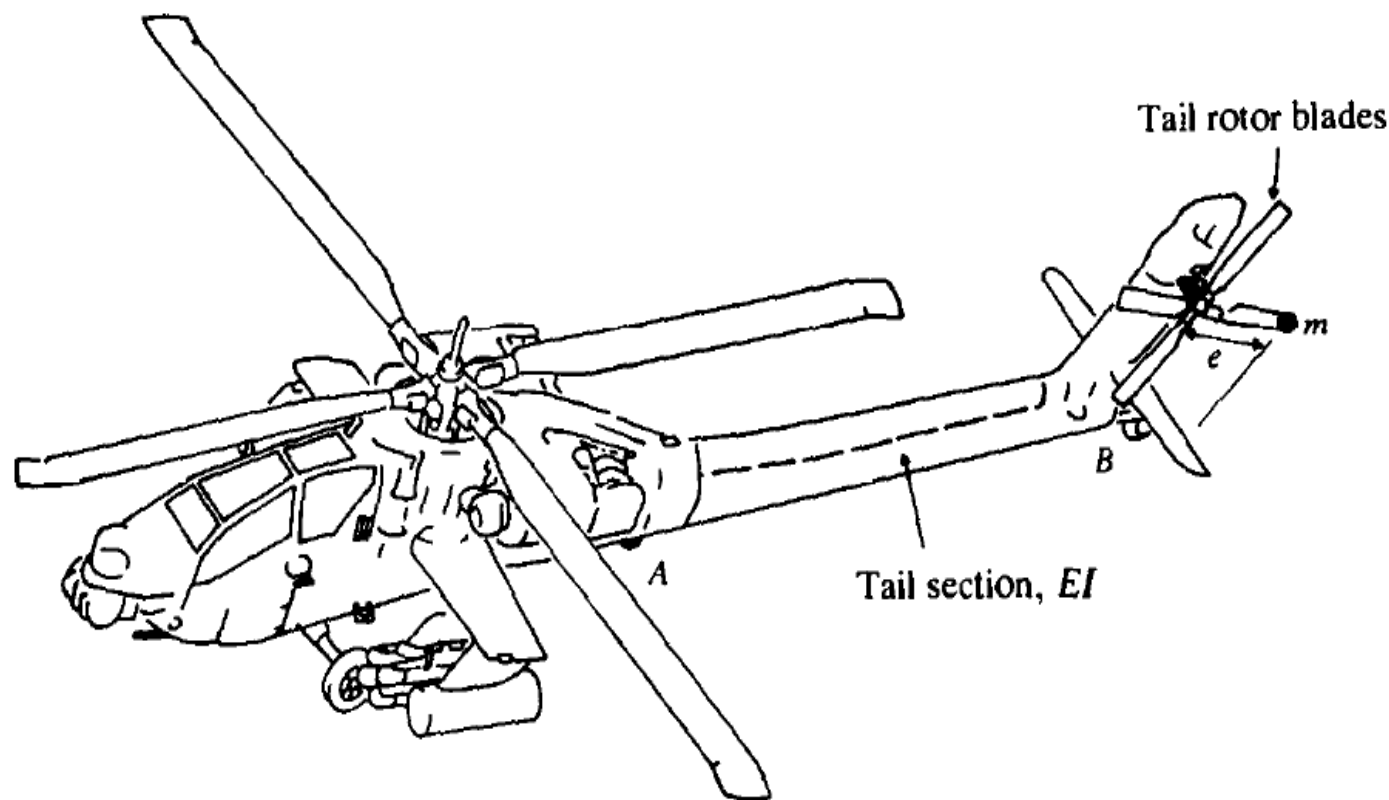
Steady state response is given by Eq. (3.25):

$$x_p(t) = X \cos(\omega t - \phi)$$

$$\text{where } X = \frac{5}{\left[\left(112.5 - 11 (20^2) \right)^2 + \left(50 (20) \right)^2 \right]^{\frac{1}{2}}} = 0.001136 \text{ m}$$

$$\text{and } \phi = \tan^{-1} \left(\frac{50 (20)}{112.5 - 11 (20^2)} \right) = -0.2291 \text{ rad} = -13.1287^\circ$$

One of the tail rotor blades of a helicopter has an unbalanced mass of $m = 0.5 \text{ kg}$ at a distance of $e = 0.15 \text{ m}$ from the axis of rotation, as shown in Fig. 3.29. The tail section has a length of 4 m , a mass of 240 kg , a flexural stiffness (EI) of $2.5 \text{ MN} \cdot \text{m}^2$, and a damping ratio of 0.15 . The mass of the tail rotor blades, including their drive system, is 20 kg . Determine the forced response of the tail section when the blades rotate at 1500 rpm .

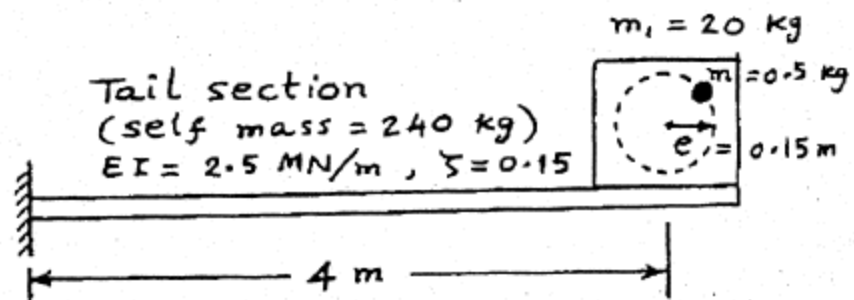


k = spring constant of cantilever beam

$$= \frac{3EI}{l^3} = \frac{3(2.5 \times 10^6)}{4^3}$$

$$= 0.1172 \times 10^6 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m_1 + 0.25 m_b}} = \sqrt{\frac{0.1172 \times 10^6}{20 + 0.25(240)}} = 38.2753 \text{ rad/sec}$$



$$\omega = 2\pi(1500)/60 = 157.08 \text{ rad/sec}$$

$$r = \omega/\omega_n = 157.08/38.2753 = 4.1040, \quad r^2 = 16.8428$$

Forced response is given by Eq. (3.79):

$$x_p(t) = X \sin(\omega t - \phi)$$

where

$$\begin{aligned} X &= \frac{m e}{m_1} \cdot \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \\ &= \frac{(0.5)(0.15)}{20} \cdot \frac{16.8428}{\sqrt{(1-16.8428)^2 + (2 \times 0.15 \times 4.1040)^2}} \\ &= 3.9747 \times 10^{-3} \text{ m} = 3.9747 \text{ mm} \end{aligned}$$